PHYSICAL REVIEW E VOLUME 51, NUMBER 2 FEBRUARY 1995

Domains and synchronization in high-dimensional cellular automata

P.-M. Binder
Center for Nonlinear Studies, Mail Stop B-258, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
*and Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501**
(Received 31 August 1994)

Difference patterns are studied in high-dimensional totalistic cellular automata. The results limit the possible mechanisms for the global (quasi)periodic behavior observed in some of these models.

PACS number(s): 05.45.+b, 47.27.Sd

A few years ago a generic argument was given against the existence of collective behavior with period larger than 2 in extended systems with local interactions [1]. Several exceptions to this argument have recently been found [2] in cellular automata (CA) models in which the average magnetization exhibits period 3 or near 3, with superimposed deterministic noise. Examples are now known in as few as three dimensions [3] and as many as six, as well as in coupled map lattice models [2].

Several attempts [2,4–6] have been made to understand the phenomenology of these models, for example in terms of second-order discrete dynamics [5], or of difference equations involving site-site correlations [6]; further studies [7,8] have addressed the possible *mechanisms* by which this puzzling collective behavior emerges. In this paper we examine the recent proposal [8,9] that the global periodicity may be a manifestation of local cyclic subsystems which do not interact [10]. Specifically, we look at difference patterns between almost identical configurations in both the three-dimensional Hemmingsson model [3], which has a collective period close to 3, and one of the original Chaté-Manneville models [2] which exhibits magnetization with period 3 plus noise. The patterns, which take on value 1 if the states of sites at the same position but in different configurations are different at a given time step, show clearly that a single spin flip can affect the values of spins anywhere in the system rapidly and uniformly, contrary to what a system made of noninteracting subsystems would do [11].

The first model considered in this work is the three-dimensional CA found by Hemmingsson [3]; each site (spin) of the automaton can take value 0 or 1; at each time step every site is updated to be 1 if the sum of the site and its six nearest neighbors is exactly 0 or 5, and to be 0 otherwise. One hundred systems of size 81^3 were simulated with periodic boundary conditions. Difference patterns were observed along the three principal lattice directions and along the four main diagonals. In all cases the changes caused by a flip of the central spin are carried with roughly constant speed [12], with value of 0.485 ± 0.02 along the principal directions. The maximum possible speed for spin change in this model would be 1 because of the nearest-neighbor interaction. A typical plot of the difference pattern along a particular row is shown in Fig. 1. Time runs vertically from the top, and space horizontally. Note that, unlike in one-dimensional CA, the pattern can be empty at some time steps; if that is the case, differences can still be present in adjacent rows or columns. The patterns are reminiscent of CA models belonging to Wolfram's class III [13].

A four-dimensional model in which sites are updated to be 1 if the sum of the nine-site von Neumann neighborhood is between 3 and 8 was also studied. Systems of size 31×20^3 were considered, but only if the asymptotic period 3 was found. Difference patterns were only monitored along the dimension of size 31. The results are similar to those for the Hemmingsson model. We note that plots similar to Fig. 1 were obtained both for randomly generated configurations

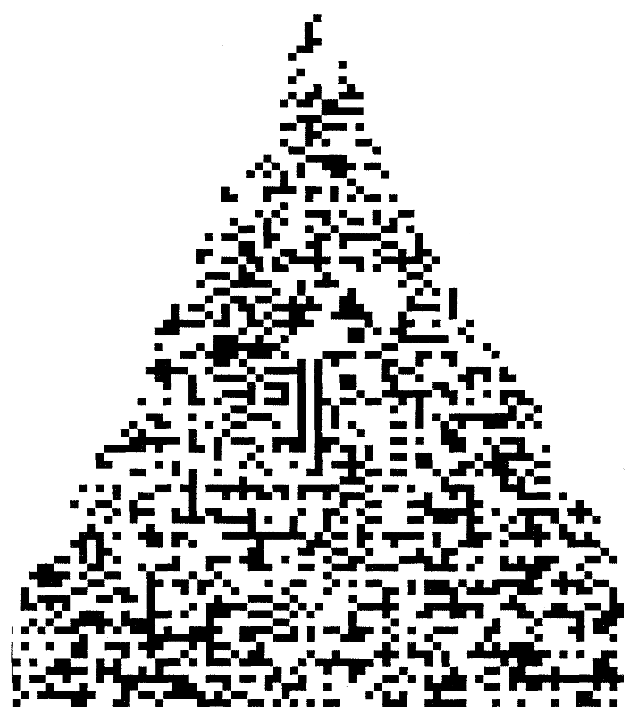

FIG. 1. Difference pattern along a principal direction in the Hemmingsson model [3]. Horizontal axis: space; vertical axis: time, running from the top. Dark pixels indicate that spins in two initially almost identical configurations have different values. System size is 81^3. Speed of information transmission is about 0.485. Difference patterns grow uniformly in every direction, indicating that no domain walls are present.

*Mailing address.

and for states reached after a suitable transient time had elapsed (typically 5000 time steps).

The simulations in this paper show beyond doubt that the instances of (quasi)periodic collective behavior observed in cellular automata of dimensions 3 or greater are not caused by independent cyclic subsystems, but rather by global synchronization. If the former were the case, the difference pattern between nearly identical configurations would be constrained in at least one direction, contrary to what we observe. The conclusion is that the results in [8] are not relevant to understanding the mechanisms governing the high-dimensional models in [2–4]. For example, in the one-dimensional rule 73 studied in [8] the spin sequence (0110) is impervious to spin values to its left or right, and therefore confines the propagation of spin flips. This is confirmed, for example, by damage spreading simulations reported in [14], and is contrary to the observations of the present paper. Conversely, the damage spreading results presented here preclude the existence of domain walls (and hence of isolated subsystems) such as the ones found in the one- and two-dimensional models studied in [8].

The author would like to thank S. Wolfram for useful discussions. The work at Los Alamos was performed under the auspices of the U.S. Department of Energy. The work at the Santa Fe Institute was supported in part by grants from the John D. and Catherine T. MacArthur Foundation, the National Science Foundation, and the U.S. Department of Energy.

[1] T. Bohr, G. Grinstein, Y. He, and C. Jayaprakash, Phys. Rev. Lett. **58**, 2155 (1987); C. H. Bennett, G. Grinstein, Y. He, C. Jayaprakash, and D. Mukamel, Phys. Rev. A **41**, 1932 (1990).
[2] H. Chaté and P. Manneville, Europhys. Lett. **14**, 409 (1991); Prog. Theor. Phys. **87**, 1 (1992).
[3] J. Hemmingsson, Physica A **183**, 255 (1992).
[4] J. A. C. Gallas, P. Grassberger, H. Herrmann, and P. Ueberholz, Physica A **180**, 19 (1992).
[5] P.-M. Binder and V. Privman, Phys. Rev. Lett. **68**, 3830 (1992).
[6] J. Hemmingsson and H. J. Herrmann, Europhys. Lett. **23**, 15 (1993).
[7] Y. Pomeau, J. Stat. Phys. **70**, 1379 (1993).
[8] J. Hemmingsson, A. Sørensen, H. Flyvbjerg, and H. J. Herrmann, Europhys. Lett. **23**, 629 (1993).
[9] J. Gorodkin, A. Sørensen, and O. Winther, Complex Syst. **7**, 1 (1994).
[10] The explanations of Refs. [7,8] are only superficially related; Pomeau actually allows weak coupling between mesoscopic subsystems.
[11] Unless a spin flip accidentally destroys an otherwise stable domain wall, the difference pattern of a collection of noninteracting subsystems should stop growing in at least one direction.
[12] Constantly growing difference patterns are often associated with positive spatial Lyapunov exponents.
[13] S. Wolfram, Rev. Mod. Phys. **55**, 601 (1983).
[14] *Theory and Applications of Cellular Automata*, edited by S. Wolfram (World Scientific, Singapore, 1986).